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FAST TRACK COMMUNICATION

Parity-odd effects and polarization rotation in graphene

I V Fialkovsky[1,2] and D V Vassilevich[2,3]

[1] Insituto de Física, Universidade de São Paulo, SP, Brazil
[2] Department of Theoretical Physics, St Petersburg University, Russia
[3] CMCC, Universidade Federal do ABC, Santo André, SP, Brazil

E-mail: dvassil@gmail.com

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Abstract
We show that the presence of parity-odd terms in the conductivity (i.e. in the polarization tensor of Dirac quasiparticles in graphene) leads to the rotation of polarization of the electromagnetic waves passing through suspended samples of graphene. Parity-odd Chern–Simons-type contributions appear in external magnetic fields, giving rise to a quantum Faraday effect (though other sources of parity-odd effects may also be discussed). The estimated order of the effect is well above the sensitivity limits of modern optical instruments.

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Since the experimental discovery of graphene [1], its unusual properties have been increasingly attracting attention of researchers, see the reviews in [2]. Quantum field theory proved to be a very successful approach to the description of the dynamics of elementary excitations in graphene, which are supposed to be $(2+1)$-dimensional fermions [3]. It is very well known that, in the massless case, such fermions exhibit the so-called parity anomaly[4], which leads in particular to the presence of the Chern–Simons term in the effective action for an external electromagnetic field. Other parity-violating terms, which can arise in the massive case $m \neq 0$, enter the effective action multiplied by the odd powers of m. In graphene, the fermions are doubled, and the corresponding gamma-matrices are taken in two inequivalent representations (differing by an overall sign). It is possible therefore to adjust the parameters of the model (couplings of the fermions to external field, etc), and the rules of analytic continuation of quantum determinants in such a way that all parity-odd terms in the complete effective action are precisely cancelled. Such a cancellation, even if it happens, cannot be universal. Recent works [7] on the quantum Hall effect in graphene show that the Chern–Simons term must appear at least in a constant magnetic field. This corresponds to the non-diagonal (parity-odd) part of the dynamical conductivity of graphene in an external magnetic field found in [8, 9].

[4] For the first time the induced Chern–Simons term was calculated by Redlich [4]. Later on, many issues regarding the parity anomaly, especially at finite temperature, were clarified in [5]. For an overview, see [6].

One may even suppose that such effects also appear under more general conditions. It is natural to look for their physical manifestations.

In the present communication, we argue that the parity-odd (quantum) effects should lead to the polarization rotation of the electromagnetic waves passing through suspended samples of mono- and few-layer graphene films. Below, we shall relate the polarization rotation angle to the real part of the parity-odd one-loop two-photon amplitudes. We shall also estimate the order of the effect and discuss the physical conditions under which this effect may be measurable. It will definitely happen in an external magnetic field thus giving rise to the quantum Faraday effect, but the polarization rotation without a magnetic field cannot be excluded. We stress that both negative and positive results of an experiment measuring such rotation will deliver valuable information on the structure of graphene. To get an idea of how large or small the effect could be we shall consider the simplest model of a single massive fermion. We emphasize that the P-odd contributions appearing in the polarization operator, or, equivalently, the additional terms in the non-diagonal part of dynamical conductivity, do not influence any predictions for the quantum Hall effect in graphene since they disappear in the dc limit ($\omega \to 0 \Leftrightarrow m \to \infty$). On the other hand, parity-odd conductivity of any nature unavoidably leads to polarization rotation effects, as is evident from the calculations given below.

The mechanism of creation and physical manifestation of the Chern–Simons-like contributions to the effective action is very similar to those proposed for four-dimensional QED [10], although in four dimensions the effects appear at much larger (astrophysical) scales.

Optical polarization effects in graphite and in arrays of carbon nanotubes has been subject to extensive study, see e.g. [11]. They were attributed to the anisotropy of the two-dimensional Brillouin zone of graphite or, equivalently, the anisotropy of the graphene crystal lattice. Similar ideas were used to predict the orientation dependence of the absorption of high-frequency light in graphene [12]. The physics behind these effects is completely different from what we shall consider below.

Let us consider a model, or rather a class of models. We assume that the propagation of fermions is described by a (2 + 1)-dimensional action

$$S_\psi = \int \mathrm{d}^3x\, \bar\psi\, (\mathrm{i}\tilde{\partial\!\!\!/} - e\not{A} + \cdots)\, \psi, \tag{1}$$

where

$$\tilde{\partial\!\!\!/} \equiv \tilde\gamma^l \partial_l, \quad l = 0, 1, 2 \quad \tilde\gamma^0 \equiv \gamma^0, \quad \tilde\gamma^{1,2} \equiv v_F \gamma^{1,2}, \quad \gamma_0^2 = -(\gamma^1)^2 = -(\gamma^2)^2 = 1. \tag{2}$$

The Dirac matrices γ^l may be taken in a reducible representation. v_F is the Fermi velocity. In our units, $\hbar = c = 1$, $v_\mathrm{F} \simeq (300)^{-1}$. The dots in (1) denote other parameters, such as the mass or chemical potential; see [13] for a complete list of allowed interactions and explanations of their physical meaning. For us it is only important that the interaction with the electromagnetic field A be gauge invariant. The electromagnetic part of the action we choose as $S_\mathrm{EM} = -1/4 \int \mathrm{d}^4x F_{\mu\nu}^2$, $\mu, \nu = 0, 1, 2, 3$. In these units $e^2 = 4\pi\alpha \simeq 4\pi/137$.

After integrating out the fermions, one arrives at the following effective action for the electromagnetic field in the quadratic approximation:

$$S_\mathrm{eff} = \frac{1}{2} \int \mathrm{d}^3x\, \mathrm{d}^3y\, A_j(x) \Pi^{jl}(y - x) A_l(y), \tag{3}$$

where, after the Fourier transform,

$$\Pi^{mn} = \frac{\alpha}{v_\mathrm{F}^2} \eta_j^m \left[\Psi(\tilde p) \left(g^{jl} - \frac{\tilde p^j \tilde p^l}{\tilde p^2} \right) + \mathrm{i}\phi(\tilde p) \epsilon^{jkl} \tilde p_k \right] \eta_l^n \tag{4}$$

with $\epsilon^{012} = 1$, $\eta_j^n = \mathrm{diag}(1, v_F, v_F)$, $\tilde{p}^m \equiv \eta_n^m p^n$. The functions Ψ and ϕ are model-dependent and can take complex values. Some more comments regarding formula (4) are in order. From (2) it is clear that, due to the presence of effective two-dimensional speed of light v_F, quantum corrections will depend on the rescaled momentum $\tilde{p}$ rather than on p. This is reflected in the construction in square brackets of (4) which reproduces the standard tensor structure of the polarization tensor, but with p substituted by $\tilde{p}$. The multiplier v_F^{-2} appears due to the relation $\mathrm{d}^3 q = v_F^{-2}\,\mathrm{d}^3\tilde{q}$ for the integration measure of the loop momentum. The overall rescalings η_j^m of the polarization operator appear since the electromagnetic potential is also multiplied by the rescaled gamma-matrices (2). It is worth mentioning that the polarization tensor is transversal with respect to the unrescaled momenta, $p_n \Pi^{nm}(p) = 0$, as expected from the gauge invariance of the model.

The precise form of the functions $\Psi(p)$ and $\phi(p)$ has been calculated in various models. The function Ψ was for the first time calculated in the framework of a three-dimensional QED in [14]. In connection with graphene systems it was presented in [15] for a semi-relativistic reduced QED model containing three parameters (mass gap, chemical potential and temperature). Later it was generalized in [8] to include the scattering rate and external magnetic field, and recently rederived in [16]. Predictions for the absorption of light in the visible part of the spectra based on the calculations of the even part of the polarization function (or, equivalently, of the dynamical conductivity [9]) are in good agreement with the experimental data [17]. The parity-odd part of the polarization tensor for planar fermions was calculated in [4, 5]. Chern–Simons modifications of the conductor boundary conditions were considered in [18]. The calculations of the Casimir force, due to the Chern–Simons action on a surface (without a parity-even part), were done in [19].

Let us embed the surface occupied by the graphene sample in the (3 + 1)-dimensional Minkowski space by identifying it with the $x^3 \equiv z = 0$ plane. The propagation of electromagnetic waves is described by Maxwell's equations with a delta-function interaction corresponding to (3)

$$\partial_\mu F^{\mu\nu} + \delta(z)\Pi^{\nu\rho}A_\rho = 0, \tag{5}$$

where we extended the polarization operator to a 4×4 matrix by setting $\Pi^{3\mu} = \Pi^{\mu 3} = 0$, $\mu, \nu = 0, 1, 2, 3$. Due to the delta-function interaction in (5) the following matching condition must be imposed on the field A_μ:

$$\begin{aligned} &A_\mu|_{z=+0} = A_\mu|_{z=-0} \\ &(\partial_z A_\mu)_{z=+0} - (\partial_z A_\mu)_{z=-0} = \Pi_\mu{}^\nu A_\nu\big|_{z=0}. \end{aligned} \tag{6}$$

Let us consider a solution in the form of a plane wave propagating along the z-axis from $z = -\infty$ with the initial polarization parallel to $x^1 \equiv x$, which is being reflected by and transmitted through the graphene sample:

$$A = \mathrm{e}^{-\mathrm{i}\omega t} \begin{cases} \mathbf{e}_x\,\mathrm{e}^{\mathrm{i}k_3 z} + (r_{xx}\mathbf{e}_x + r_{xy}\mathbf{e}_y)\,\mathrm{e}^{-\mathrm{i}k_3 z}, & z < 0 \\ (t_{xx}\mathbf{e}_x + t_{xy}\mathbf{e}_y)\,\mathrm{e}^{\mathrm{i}k_3 z}, & z > 0 \end{cases} \tag{7}$$

where $\mathbf{e}_{x,y}$ are unit vectors in the direction $x^{1,2}$. The on-shell condition (free Maxwell equations outside $z = 0$) implies $k_3 = \omega$. For such waves the matching conditions (6) simplify

$$\begin{aligned} &A_a|_{z=+0} = A_a|_{z=-0} \\ &(\partial_z A_a)_{z=+0} - (\partial_z A_a)_{z=-0} = \alpha\left[\Psi(k)\delta_a^b + \mathrm{i}\omega\phi(k)\epsilon_a{}^b\right]A_b, \end{aligned} \tag{8}$$

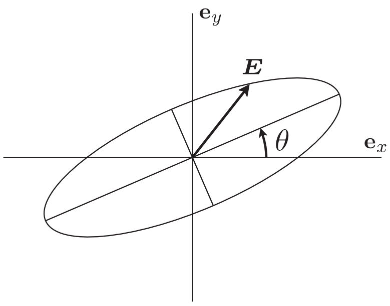

Figure 1. Polarization of the transmitted wave.

where $a, b = 1, 2$, $\epsilon_1{}^2 = -\epsilon_2{}^1 = 1$. The transmission and reflection coefficients can be found relatively easily:

$$\begin{aligned} t_{xx} &= \frac{-2\omega(\mathrm{i}\alpha\Psi + 2\omega)}{\alpha^2\Psi^2 - 4\mathrm{i}\alpha\omega\Psi - (4+\alpha^2\phi^2)\omega^2}, \\ t_{xy} &= \frac{2\phi\alpha\omega^2}{\alpha^2\Psi^2 - 4\mathrm{i}\alpha\omega\Psi - (4+\alpha^2\phi^2)\omega^2}, \\ r_{xx} &= t_{xx} - 1, \qquad r_{xy} = t_{xy}. \end{aligned} \tag{9}$$

The amplitude of the transmitted wave reads

$$\mathcal{A} = \sqrt{|t_{xx}|^2 + |t_{xy}|^2} \simeq 1 - \left|\frac{\operatorname{Im}\Psi}{2\omega}\right| \alpha + O(\alpha^2), \tag{10}$$

meaning that there is an absorption in the model at the linear order in α, which is in agreement with experiment [17]. To all orders of α the flux conservation condition $|r_{xx}|^2 + |r_{xt}|^2 + |t_{xx}|^2 + |t_{xy}|^2 = 1$ holds if both Ψ and ϕ are real.

Let us study the polarization rotation. In the generic case the transmitted wave has an elliptic polarization, see figure 1. The ratio of the semi-axes of the ellipse is given by

$$\begin{aligned} R &= \frac{|s_+| - |s_-|}{|s_+| + |s_-|} \simeq -\frac{\alpha \operatorname{Im}\phi}{2} + O(\alpha^2) \\ s_+ &= \frac{t_{xx} - \mathrm{i}t_{xy}}{2}, \qquad s_- = \frac{t_{xx} + \mathrm{i}t_{xy}}{2} \end{aligned} \tag{11}$$

while the angle with $\mathbf{e}_x$ is

$$\theta = \frac{1}{2}\arg\frac{s_+}{s_-} = -\frac{1}{2}\arg\frac{\mathrm{i}\,\alpha\Psi + 2\omega + \mathrm{i}\,\phi\alpha\omega}{\mathrm{i}\,\alpha\Psi + 2\omega - \mathrm{i}\,\phi\alpha\omega} = -\frac{\alpha\operatorname{Re}\phi}{2} + O(\alpha^2). \tag{12}$$

Let us further note that

$$t_{xy} = \frac{-\alpha\omega\phi}{\mathrm{i}\alpha\Psi + 2\omega} t_{xx}. \tag{13}$$

The transmitted wave is linearly polarized if the ratio t_{xx}/t_{xy} is real. This happens exactly if $\operatorname{Re}\Psi = \operatorname{Im}\phi = 0$, but in the leading order of α it is enough to require $\operatorname{Im}\phi = 0$. The angle between the electric field of the transmitted wave and $\mathbf{e}_x$ is still given by (12).

To estimate the order of magnitude of the polarization rotation effects let us consider the simple case of a single massive fermion in 2 + 1 dimensions. The functions Ψ and ϕ in (4) can be calculated explicitly:

$$\Psi(p) = \frac{2m\tilde{p} - (\tilde{p}^2 + 4m^2)\mathrm{arctanh}(\tilde{p}/2m)}{2\tilde{p}}, \tag{14}$$

$$\phi(p) = \frac{2m\,\mathrm{arctanh}(\tilde{p}/2m)}{\tilde{p}} - 1 \tag{15}$$

where $\tilde{p} \equiv +\sqrt{\tilde{p}_j \tilde{p}^j}$, and we assume $m \geqslant 0$. Both (14) and (15) are consistent with earlier (and more general) computations, see [4–6, 8, 14–16]. We recall that the term -1 in (15) is a result of the Pauli–Villars subtraction which is necessary to restore full gauge invariance of the effective action [4].

Along with more elaborate couplings and larger number of parameters in realistic models of graphene, number of fermionic components is larger. Therefore, the actual values of the functions Ψ and ϕ may change significantly. We believe that the formulae are enough to make an order of magnitude estimate of the expected effects.

For normal incident waves, $\tilde{p} = \omega$. Both Ψ and ϕ are real for $\omega < 2m$ and complex for $\omega > 2m$ yielding to absorption which starts at $\omega = 2m$. The absorption and polarization rotation effects are small (so that one can use an α-expansion) everywhere except for a narrow region around $\omega = 2m$. At $\omega = 2m$ the angle θ has a peak with a maximum value of $\pi/4$. It is likely, however, that this large effect will be smeared out in a more realistic model. At low frequencies, $\omega \ll 2m$, the function ϕ vanishes and the polarization rotation effects become negligible.

At large frequencies, $\omega \gg 2m$, $\Psi \simeq \mathrm{i}\pi\tilde{p}/2$ and $\phi \simeq -1$. Taking into account the actual number of fermion species used for the correct description of graphene quasi-particles, see [8], one finds that equation (10) in the high frequency regime is in perfect agreement with experimental results [17]. On the other hand, the rotation angle in this limit is $\theta \simeq \alpha/2$ which is well above the sensitivity limit of modern optical devices. Phenomenologically acceptable values of the mass parameter (energy gap, chemical potential, etc) for many realistic systems are at maximum order of 1 eV. Therefore, if a parity-odd part of the effective action exists, the polarization rotation effects must be readily detectable in the visible part of the spectrum.

Under which conditions could one expect the above-described effects to take place? The first candidate to consider is a classically P-even system in an external magnetic field. In this case, the presence of the Chern–Simons term and, consequently, of the polarization rotation, seems to be unavoidable. Indeed, the effective action for fermions in graphene, in the presence of a constant magnetic field, does contain a Chern–Simons term for this field [7]. On the other hand, direct calculations of the polarization operator in the external magnetic field (made in [8] for the case of vanishing spatial part of external momenta) shows explicitly the Chern–Simons contribution (compare the RHS of (8) effectively defining the conductivity in our model with (A4), (A8) of [8]). The polarization rotation, due to parity-odd quantum effects in the external magnetic field, is a kind of *quantum Faraday effect*.

An even more exciting (though a bit more speculative) possibility is that the parity-violating terms might appear without a constant external magnetic field. Such a possibility was studied in the pioneering paper by Haldane [20]. Note that even a relatively tiny non-cancellation of parity-odd parts of the effective action between different fermion species, as small as 10% of (15), should already be detectable with up-to-date experimental techniques. In addition, there are two-dimensional systems without reflection symmetry, like the few-layer graphene films [21], where there is no particular reason to expect this cancellation.

To summarize, there are various arguments in favour and against the existence of parity-odd quantum effects in graphene and other two-dimensional systems in condensed matter physics leading to Chern–Simons-type contributions to the effective action. This situation may be resolved by simple experiments with polarized light transmitted through suspended films. Any result of such experiments, positive or negative, will give much information about the electron structure of graphene and similar systems.

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